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ESTIMATION OF TIME DEPENDENT PARAMETERS IN LINEAIR MODELS USING CROSS SECTIONS, PANELS OR BOTH

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ESTIMATION OF TIME DEPENDENT PARAMETERS IN LINEAR MODELS USING CROSS SECTIONS, PANELS OR BOTH

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Abstract

In this note we consider the estimation of time dependent parameters in linear models from panel data, cross sections or both. We determine the fraction of individuals that should be reinterviewed each period in order to minimize the variance of the most efficient estimator of linear combinations of the parameters. Moreover we derive simple sufficient conditions for the optimal fraction to be zero or one respectively.

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1. Introduction

In recent years economists often have financial resources at their disposal to have data collected. In this note we analyse how to spend this money efficiently if the aim is e.g. to monitor average expenditures on some consumption categories by either interviewing the same individuals in several periods or interviewing different individuals in different periods or a combination of these two approaches. The first approach yields a data set known as a panel, while the second approach gives a series of cross sections. Recently it has been stressed in the econometric literature that panel data are not indispensible for the identification of parameters in a wide class of models (see e.g. Deaton [1985] and Heckman and Robb [1985a, 1985b]). Little attention however seems to have been paid to the analysis of the efficiency of estimates obtained from panels or cross sections.

In this note we concentrate on the estimation of linear combinations $\xi'\mu = \Sigma_{t=1}^{T}\xi_t\mu_t$ of the period means μ_t in the simple analysis of variance model

$$y_{it} = \mu_t + \alpha_i + \varepsilon_{it} \qquad (i = 1,\dots,N; \quad t = 1,\dots,T) \tag{1}$$

where the α_i and ε_{it} are i.i.d. normal random variables with mean zero and variances σ_α^2 and σ_ε^2 respectively which are mutually independent and independent of the unknown constants μ_t. Moreover we discuss extensions to the analysis of covariance model

$$y_{it} = \mu_t + \beta_t x_{it} + \alpha_i + \varepsilon_{it} \tag{2}$$

where the x_{it} are observed and independent of α_i and ε_{it} and to (1) or (2) with linear restrictions on the time dependent parameters. Throughout this paper we assume for simplicity that the parameters σ_α^2 and σ_ε^2 are known a priori. If these parameters are unknown and replaced by consistent estimates the same results hold true asymptotically.

Let η denote the relative cost of interviewing T different individuals in T periods compared to interviewing the same individual T times. The value of η of course depends on the problem under consideration but experts suggest that it will usually be slightly larger than one. The only formal analysis of η in the literature we are aware of is presented by Duncan, Juster and Morgan [1987] who suggest that $1.3 < \eta < 1.7$. We show in section two that a pure panel will yield the most efficient estimate of any linear combination of the period means in (1) if $\eta > 1 + (T-1)\rho$ with $\rho = \sigma_\alpha^2(\sigma_\alpha^2 + \sigma_\varepsilon^2)^{-1}$ while the same holds true for pure cross sections if $\eta < 1-\rho$. If one is estimating changes in means the condition for optimality of panels can be relaxed to $\eta > 1-\rho$ while in case of an estimate of the average mean cross sections are already optimal if $\eta < 1 + (T-1)\rho$. Analytical and numerical results are presented for cases in which neither of these conditions is satisfied. In section three a numerical illustration is given using Dutch consumer expenditure data. Extensions to the analysis of covariance model (2) are provided in section four, while section five contains some concluding remarks.

2. Analytical and numerical results for the analysis of variance model

Denote the maximum sample-size per period, given the available funds, if different individuals are interviewed each period by N and the fraction of the funds used to collect panel data by λ, which implies that the first $\lambda\eta N$ individuals will be reinterviewed every period while the remaining $(1-\lambda)N$ individuals will be replaced each period. The analysis of this type of data is advocated e.g. by Kish [1986] who refers to it as a split panel design (SPD). We will determine the optimal value of λ as a function of η, ρ and the linear combination of μ_t in (1) one is interested in. A similar analysis of the choice between pre-experimental observations and control groups in social experimentation has been presented by Aigner and Balestra [1987].

It is well known (see e.g. Hsiao [1986, p. 34 ff.]) that the efficient estimator of $\mu' = (\mu_1,\ldots,\mu_T)$ in (1) using only the panel part of the data is the Aitken estimator $\hat{\mu}_p$ (which is in this case identical to the OLS estimator) and that

$$\hat{\mu}_p \sim N(\ \mu\ ,\ (\lambda\eta N)^{-1}V_p) \tag{3}$$

with $V_p = \sigma_\varepsilon^2 I_T + \sigma_\alpha^2 \iota_T\iota_T'$ and ι_T is a T dimensional column vector of ones. Analogously the efficient estimator based on the cross section information only is the OLS estimator $\hat{\mu}_{cs}$ for which

$$\hat{\mu}_{cs} \sim N(\ \mu\ ,\ ((1-\lambda)N)^{-1}V_{cs}) \tag{4}$$

with $V_{cs} = (\sigma_\varepsilon^2+\sigma_\alpha^2)I_T$. Since $\hat{\mu}_p$ and $\hat{\mu}_{cs}$ are independent the efficient estimator which uses all available data is given by

$$\hat{\mu} = \{\lambda\eta V_p^{-1} + (1-\lambda)V_{cs}^{-1}\}^{-1}\{\lambda\eta V_p^{-1}\hat{\mu}_p + (1-\lambda)V_{cs}^{-1}\hat{\mu}_{cs}\}. \tag{5}$$

It is easily verified that

$$\xi'\hat{\mu} \sim N(\ \xi'\mu\ ,\ N^{-1}\xi'[\ V_{cs}^{-1} + \lambda\ W\ ]^{-1}\xi), \tag{6}$$

where $W = \eta V_p^{-1} - V_{cs}^{-1}$. Since V_{cs}^{-1} is positive definite and W is symmetric there exists a nonsingular matrix Q such that $Q'V_{cs}^{-1}Q = I_T$ and $Q'WQ = D$ with D a diagonal matrix containing the eigenvalues d_t of $V_{cs}W$ and Q containing the (suitably normalized) eigenvectors of $V_{cs}W$ (see e.g. Gantmacher [1959, p.313 ff.]). Therefore the variance of $\xi'\hat{\mu}$ can be written as

$$\begin{aligned} \mathrm{Var}(\xi'\hat{\mu}) &= N^{-1}\xi'[V_{cs}^{-1} + \lambda W]^{-1}\xi \\ &= N^{-1}\delta'[I_T + \lambda D]^{-1}\delta = N^{-1}\sum_{t=1}^{T}\frac{\delta_t^2}{(\lambda d_t+1)} \end{aligned} \tag{7}$$

with $\delta = Q'\xi$. Straightforward algebra shows that in our case

$$V_{cs}W = (1-\rho)^{-1}[\ (\eta+\rho-1)I_T - \eta\rho\{1+(T-1)\rho\}^{-1}\iota_T\iota_T'\ ], \tag{8}$$

with eigenvalues $d_t = \eta(1-\rho)^{-1}-1 = d\ (t=1,\dots,T-1)$ and $d_T = \eta[1+(T-1)\rho]^{-1}-1$. Using the equality of the first T-1 eigenvalues we obtain

$$\mathrm{Var}(\xi'\hat{\mu}) = N^{-1}\sum_{t=1}^{T-1}\frac{\delta_t^2}{\lambda d_t+1} + N^{-1}\frac{\delta_T^2}{\lambda d_T+1} = N^{-1}\delta'\delta\left\{\frac{1-\omega}{\lambda d+1} + \frac{\omega}{\lambda d_T+1}\right\} \qquad (9)$$

with $\omega = \delta_T^2(\delta'\delta)^{-1}$. Because $V_{cs} = (\sigma_\alpha^2 + \sigma_\varepsilon^2)\, I_T = Q'Q$ and $(\sigma_\alpha^2 + \sigma_\varepsilon^2)^{-1}\iota_T/\surd T$ is the eigenvector of $V_{cs}W$ associated with d_T, (9) can finally be rewritten as

$$\mathrm{Var}(\xi'\hat{\mu}) = N^{-1}(\sigma_\alpha^2 + \sigma_\varepsilon^2)\ \xi'\xi\left\{\frac{1-\omega}{\lambda d+1} + \frac{\omega}{\lambda d_T+1}\right\} \qquad (10)$$

with $\omega = T^{-1}(\xi'\iota_T)^2/\xi'\xi$.

For the special case where $T = 2$ and $\eta = 1$ it can be easily checked from this expression that $\hat{\mu}_1 + \hat{\mu}_2$ has smallest variance if $\lambda = 0$ (pure cross section), that $\mu_2 - \mu_1$ is estimated most efficiently if $\lambda = 1$ (pure panel), while for estimating μ_1 or μ_2 the intermediary value $\lambda = 1 - (1 + \surd(1-\rho^2))^{-1}$ is optimal, which are well known results in the literature (see e.g. Raj [1968, p. 157] or Cochran [1977, p. 347]).

Equation (10) however generates more general results. The variance of $\xi'\hat{\mu}$ will be minimized at $\lambda = 1$ if $d_t > 0$ $(t=1,\dots,T)$, irrespective of ξ. The smallest eigenvalue of $V_{cs}W$ is $d_T = \eta[1+(T-1)\rho]^{-1}-1$ which implies that a pure panel will yield the most efficient estimate of any linear combination of the period means if $\eta > 1+(T-1)\rho$. The same holds true for pure cross sections if $d_t < 0$ $(t=1,\dots,T)$, or $\eta < 1-\rho$.

In some cases neither of the two conditions on η, the relative cost of interviewing T different individuals compared to interviewing the same individual T times, will be satisfied. If one is estimating changes in the means which implies $\xi'\iota_T = 0$ or $\omega = 0$ a pure panel will still be optimal if $\eta > 1-\rho$ because $d_t > 0$ $(t=1,\dots,T-1)$. The counterpart of this result is the case of estimating the overall mean, where ξ is proportional to ι_T in which case a cross section will be optimal as long as $\eta < 1+(T-1)\rho$.

If $1-\rho < \eta < 1+(T-1)\rho$ and ξ is not proportional to ι_T nor $\xi'\iota_T = 0$ it is more difficult to obtain analytical results. However the optimal value of λ, λ^*, can easily be determined numerically because it will either be a solution to the quadratic first order condition for a minimum of (10) or a boundary extremum because $\lambda \in [0, 1]$.

As an illustration we present the optimal percentage of people reinterviewed every period, 100 λ^*, as a function of ρ and T assuming that $\eta=1$ and that the aim is to estimate the period means as accurately as possible. Moreover we present in table 1 the relative efficiency of the estimator based on this sample compared to an estimator based on a pure panel or a pure cross section (which yield equally efficient estimators in this case).

Table 1. Values of λ for which the variance of the effient estimator of the period mean is minimized and relative efficiency compared to pure panel or cross section if η=1.

	T = 2		*T = 3*		*T = 6*		*T = 12*	
ρ	λ^*	*rel. eff.*	λ^*	*rel. eff.*	λ^*	*rel. eff.*	λ^*	*rel. eff.*
.3	.49	.98	.51	.96	.57	.93	.63	.89
.6	.44	.90	.48	.84	.56	.74	.64	.65
.9	.30	.72	.35	.59	.43	.43	.51	.32

Cochran [1977, p.351] showed that if η=1 and individuals are included in the sample for not more than two periods the percentage of reinterviews which minimizes $var(\hat{\mu}_t)$ tends to 50% if T increases, irrespective of the value of ρ. As evident from table 1, this result no longer holds in the present model. Replacing half of the sample every period was also found to be optimal if T is large, by Raj [1968, p. 162] who however assumed that $Eu_{i,t}u_{i,t-s}$ with $u_{it} = \alpha_i + \varepsilon_{it}$ is a decreasing function of s (s>0).

3. Estimates of Dutch consumer expenditures

In this section we will briefly consider the implications of the results in the previous sections for the estimation of the consumer expenditures of Dutch households. We use the 342 complete monthly observations in 1985 of the so called Expenditure Index panel conducted by INTOMART, a private marketing research agency, on two well defined consumption categories: food expenditures and expenditures on clothing (including shoes etc.). Precise definitions of these categories are available on request.

The maximum likelihood estimates of $\rho = \sigma_\alpha^2(\sigma_\alpha^2 + \sigma_\varepsilon^2)^{-1}$ in (1) for food and clothing are .76 and .25 with standard errors .005 and .002 respectively. These point estimates reflect the fact that food expenditures are relatively stable compared to expenditures on clothing. Model (1) was tested against an alternative where the ε_{it}'s were generated by a first order autoregressive process with a common autoregressive parameter γ. This model (with $\sigma_\alpha^2 = 0$) was considered in Raj [1968]. The Lagrange Multiplier test statistic against this alternative can be shown to be equivalent to N times the (non-centered) R^2 of a simple regression (see appendix). The values of this test statistic are 1.44 and 3.52 respectively which we do not take as evidence against the null. Unrestricted ML estimation of the covariance matrix of $\alpha_i+\varepsilon_{it}$, assuming only that the observations are independent over individuals, suggests that there is some heteroskedasticity in the data which we have however ignored.

The estimate $\hat{\rho} = 0.76$ for food suggests that the relative cost of interviewing different instead of the same individuals, η, should be smaller than $1-\hat{\rho} = 0.24$ for a cross section to yield estimates of every linear combination of the monthly food expenditures that are as accurate as the ones that can be obtained from a panel in which all households are retained for one year. If $\eta > [1 + (T-1)\hat{\rho}] = 9.36$ the panel will be preferable without ambiguity. For clothing these conditions are $\eta < .75$ and $\eta > 3.75$ respectively. In section two it was shown how these conditions are affected if one restricts attention to linear combinations $\Sigma_{t=1}^{12}\xi_t\mu_t$ with $\Sigma_{t=1}^{12}\xi_t = 0$ (change) or $\xi_1 = \xi_2 = \dots = \xi_{12}$ (annual average). The numerical results

Table 2. Minimum (maximum) relative cost of interviewing different individuals every period η for a pure panel (cross section) to yield efficient estimates

	change in means	*monthly mean*	*quarterly mean*	*annual mean*	*any linear combination*
value of ω	*0*	*1/12*	*1/4*	*1*	$\omega \in [0, 1]$
Food					
Panel if $\eta >$	*.24*	*7.1*	*8.9*	*9.4*	*9.4*
C.S. if $\eta <$	*.24*	*.27*	*.33*	*9.4*	*.24*
Clothing					
Panel if $\eta >$	*.75*	*1.7*	*2.6*	*3.8*	*3.8*
C.S. if $\eta <$	*.75*	*.81*	*.93*	*3.8*	*.75*

are given in table 2 where we also present the minimum (maximum) value of η for which a pure panel (cross section) will be optimal if the aim is to estimate monthly or quarterly expenditures respectively. These values can be obtained along the lines described in the previous section.

Because it is evident from (10) that the optimal percentage of households reinterviewed every period, $100\,\lambda^*$, depends on T, ρ, η and $\omega = T^{-1}(\xi'\iota_T)^2/\xi'\xi$ only, an alternative way to present the results in table 2 is to plot the values of λ^* as a function of η and ω if $T = 12$ and $\rho = \hat{\rho}$ as in figures 1 and 2. The results of table 2 can easily be reconstructed from these figures and moreover the reader can directly obtain the optimal value of λ for any linear combination of the period means he might be interested in. For comparison we have also indicated for which values of η and ω a pure panel will be more informative than a pure cross section. This is the case if $\eta > 1-\rho + T\rho\omega$, as can be easily checked from (10).

Finally, table 3 contains the optimal value of λ if monthly, quarterly or annual means or changes in means are to be estimated for three values of the relative cost factor η as well as the relative efficiency of the efficient estimator in case of optimal sample design compared to pure cross sections or pure panels. It is evident from these results that the optimal design can be substantially more informative than the extreme possibilities.

Figure 1. The optimal panel perunage (lambda) for food given relative cost (eta) and linear combination of interest (omega)

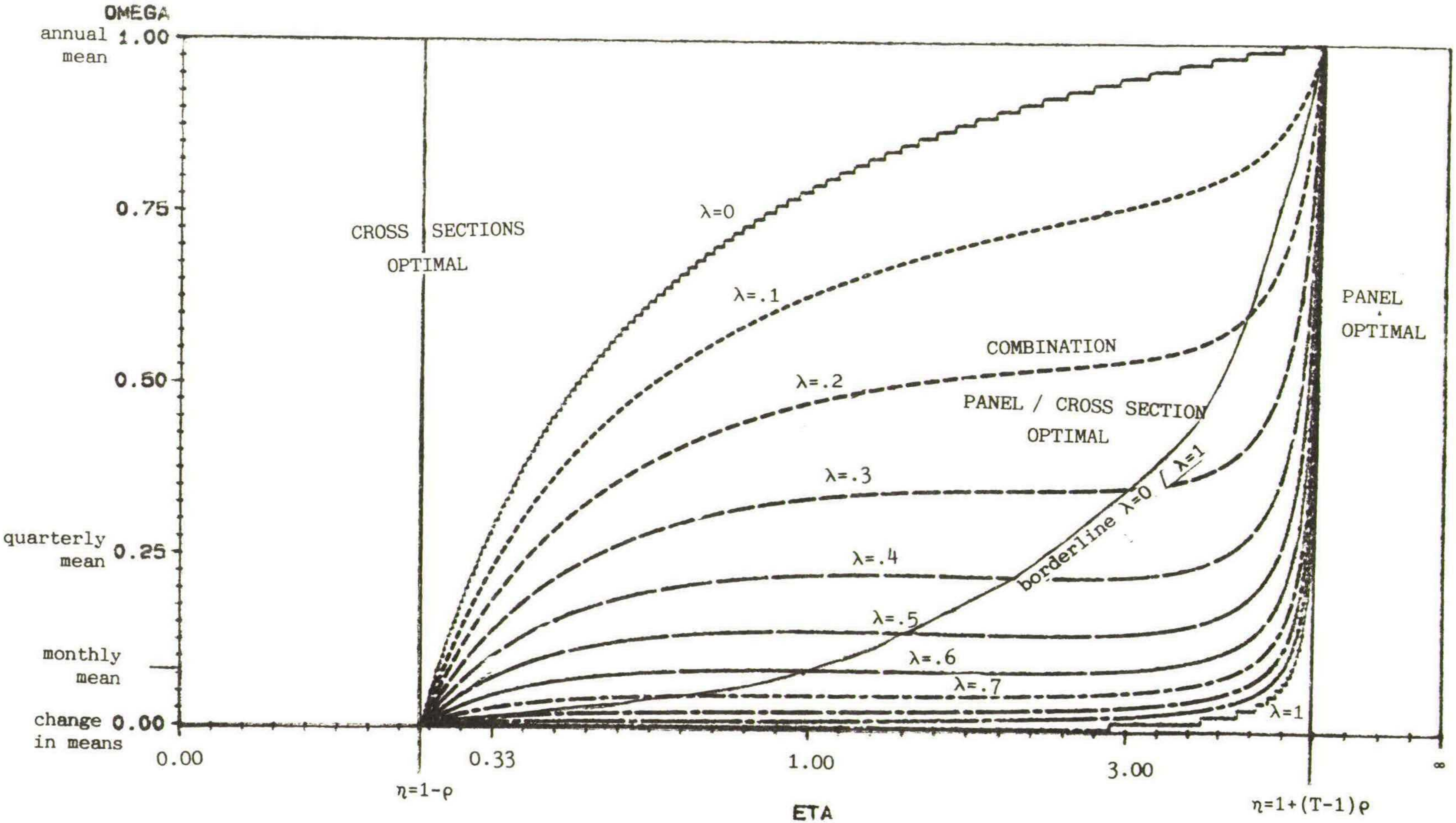

Figure 2. The optimal panel perunage (lambda) for clothing given relative cost (eta) and linear combination of interest (omega)

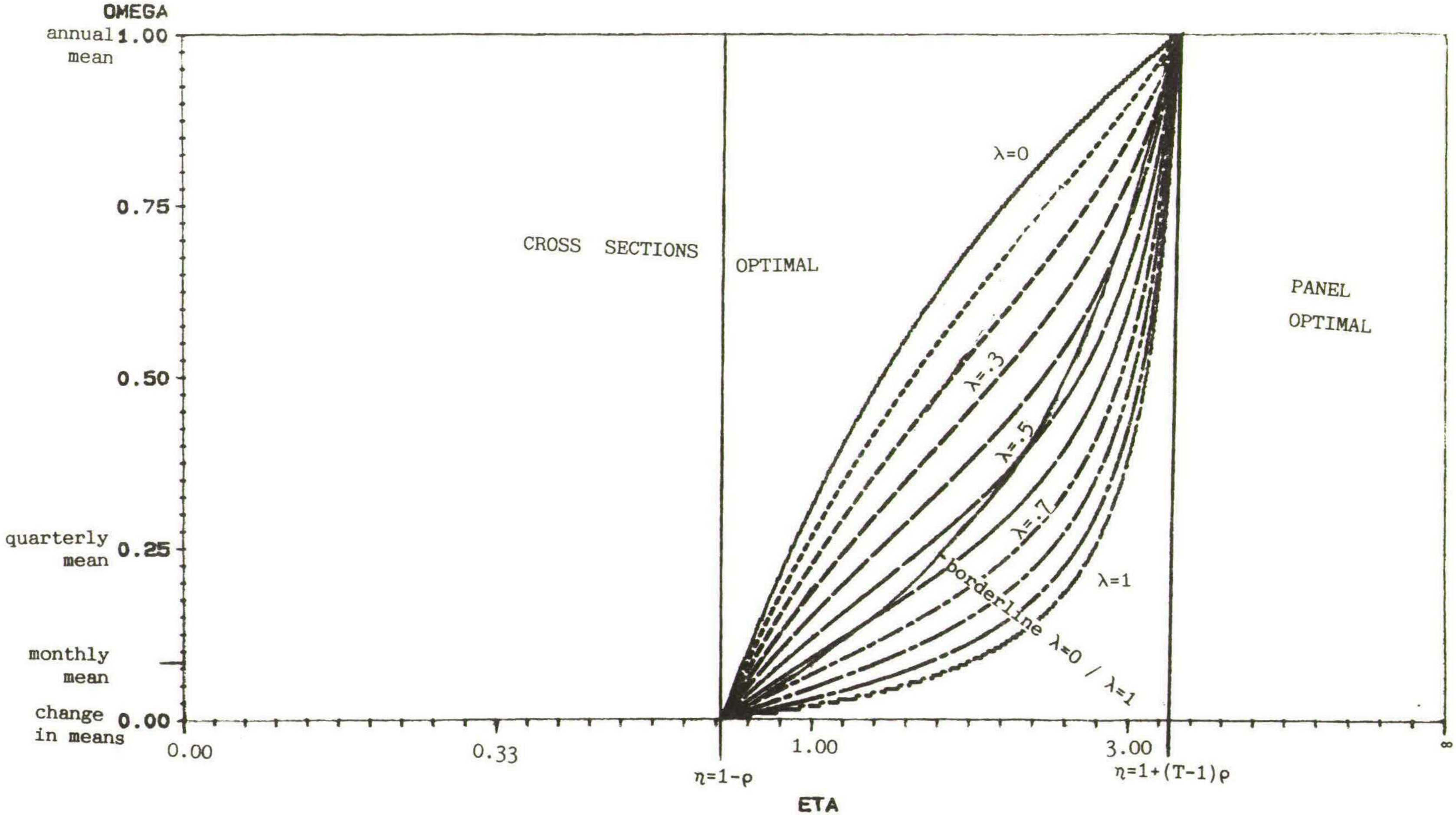

Table 3. Optimal design and relative efficiency in case of optimal sample design versus pure cross section or panel

	change in means			monthly mean			quarterly mean			annual mean		
value of ω	$\omega=0$			$\omega=1/12$			$\omega=1/4$			$\omega=1$		
relative cost η	$\eta=0.5$	$\eta=1$	$\eta=2$	$\eta=0.5$	$\eta=1$	$\eta=2$	$\eta=0.5$	$\eta=1$	$\eta=2$	$\eta=0.5$	$\eta=1$	$\eta=2$
Food												
optimal λ	1	1	1	.57	.60	.60	.30	.37	.37	0	0	0
rel. eff. w.r.t. CS	.48	.24	.12	.75	.50	.33	.92	.72	.55	1	1	1
rel. eff. w.r.t. Panel	1	1	1	.37	.50	.66	.18	.29	.44	.05	.11	.21
Clothing												
optimal λ	0	1	1	0	.63	1	0	.14	.71	0	0	0
rel. eff. w.r.t. CS	1	.75	.38	1	.91	.50	1	.99	.72	1	1	1
rel. eff. w.r.t. Panel	.67	1	1	0.50	.91	1	.33	.66	.96	.13	.27	.53

4. Extensions to an analysis of covariance model and to restricted parameters

In this section we first extend the results obtained in section 2 for the analysis of variance model (1) to the analysis of covariance model

$$y_{it} = \mu_t + \beta_t x_{it} + \alpha_i + \varepsilon_{it} \tag{2}$$

where α_i and ε_{it} are distributed as before, and are independent of the observed exogenous variable. Without loss of generality we assume that $E\, x_{it} = 0$. Subsequently, linear restrictions on the μ_t's and β_t's are incorporated into the analysis and an application to the estimation of marginal budget shares of the consumption categories analysed in section 3 is presented.

Suppose one is interested in the variance of the efficient estimator $\hat{\vartheta}$ of $\vartheta = (\mu_1, \ldots, \mu_T, \beta_1, \ldots, \beta_T)'$ given the sample design. Straightforward generalization of (7) implies that if the eigenvalues of $V_{cs}W = \eta V_{cs} V_p^{-1} - I_{2T}$, with V_{cs} and V_p the variance of $\hat{\vartheta}$ if only cross section or only panel observations are used respectively, are denoted by d_t $(t=1,\ldots 2T)$ and the corresponding matrix of eigenvectors is denoted by Q it holds true that

$$\xi'\hat{\vartheta} \sim N(\, \xi'\vartheta \,,\, N^{-1} \sum_{t=1}^{2T} \delta_t^2 \, (\lambda d_t + 1)^{-1} \,) \tag{7'}$$

with $\delta = Q'\xi$. For model (2) it can be easily checked that

$$V_{cs}W = \left[\frac{\eta}{1-\rho} - 1\right] I_{2T} - \frac{\eta}{1-\rho}\,\frac{\rho}{1+(T-1)\rho} \begin{bmatrix} \iota_T \iota_T' & 0 \\ 0 & \Omega \end{bmatrix} \tag{11}$$

with $\Omega_{ts} = E\, x_{it}x_{is} \,/\, E\, x_{it}^2$ $(t,\ s = 1,\ldots T)$.

The eigenvalues of $V_{cs}W$ are $d_t = \eta(1-\rho)^{-1}\{\, 1 - \rho[1+(T-1)\rho]^{-1} d_{xt} \,\} - 1$ with $d_{xt} = 0$ $(t=1,\ldots,T-1)$, $d_{xT} = T$ and d_{xt} $(t=T+1,\ldots,2T)$ the eigenvalues of Ω. Optimality of a pure cross section for any linear combination of μ_t's and β_t's requires $d_t < 0$ $(t=1,\ldots,2T)$ or $\eta < 1-\rho$ as in section two, since $0 < d_{xt} < T$. Similarly, a pure panel is preferable without ambiguity if the relative cost factor η satisfies $\eta > 1+(T-1)\rho$ in which case $d_t > 0$

(t=1,..,2T). If the attention is restricted to linear combinations of the β_t's only, less stringent conditons can be obtained because $\delta_t = 0$ for t=1,..,T. Therefore for any linear combination of the β_t's a pure cross section will be the optimal design if $d_t < 0$ (t=T+1,...,2T), or

$$\eta < (1-\rho)\left[1 - \rho\,\frac{d_x^{min}}{1+(T-1)\rho}\right]^{-1}, \tag{12}$$

while a panel is optimal if

$$\eta > (1-\rho)\left[1 - \rho\,\frac{d_x^{max}}{1+(T-1)\rho}\right]^{-1}, \tag{13}$$

where d_x^{min} and d_x^{max} denote the smallest and the largest eigenvalue of Ω respectively. In applications d_x^{min} and d_x^{max} can simply be estimated consistently if panel observations on x_{it} are available.

In order to obtain some feeling for these results we have considered two special cases. A first special case is the one where x_{it} can be assumed to be generated by the analysis of variance model that was discussed in the previous sections,

$$x_{it} = \mu_{xt} + \alpha_{xi} + \varepsilon_{xit} \tag{14}$$

where the α_{xi} and ε_{xit} are i.i.d. random variables with mean zero and variances $\sigma^2_{x\alpha}$ and $\sigma^2_{x\varepsilon}$ respectively which are mutually independent and independent of the unknown constants μ_{xt}, which yields $d_x^{min} = 1-\rho_x$ and $d_x^{max} = 1+(T-1)\rho_x$, with $\rho_x = \sigma^2_{x\alpha}(\sigma^2_{x\alpha}+\sigma^2_{x\varepsilon})^{-1}$. For this special case conditions (12) and (13) reduce to the earlier ones if $\rho_x = 1$. If on the other hand there is no individual effect in the exogenous variable ($\rho_x = 0$) either a pure cross section or a pure panel will be optimal because the right hand sides of (12) and (13) coincide. If neither (12) nor (13) holds the optimal value of λ can be obtained along the lines sketched in section two.

In the second special case that we consider we only assume

$$E\, x_{it} x_{is} \geq 0, \qquad s, t = 1,\dots,T \tag{15}$$

This condition appears to be satisfied for many economic variables. If (15) holds, the eigenvalues of Ω, f_t $(t=1,\dots T)$, satisfy

$$0 \leq f_t \leq 1 + (T-1) \max_{t\neq s} \Omega_{ts} \tag{16}$$

because every element of Ω is non-negative and the right hand side is the largest eigenvalue of the matrix with diagonal elements equal to 1 and off diagonal elements equal to $\max_{t\neq s} \Omega_{ts}$, which bounds every element of Ω. Using (16) it is straightforward to check that if the attention is restricted to linear combinations of the β_t's sufficient conditions for optimality of pure panels and cross sections are

$$\eta > (1-\rho) \left[1 - \rho \frac{1+(T-1)\max_{t\neq s}(\Omega_{ts})}{1+(T-1)\rho} \right]^{-1} \tag{17}$$

and

$$\eta < 1-\rho \tag{18}$$

respectively.

In applied work often a priori restrictions on the parameters in (2) such as $\beta_1 = \dots = \beta_T = \beta$ will be imposed. If the restrictions are linear such that $\varphi = R'\vartheta$ is the new set of parameters the eigenvalues of $\eta R'V_{cs}R(R'V_pR)^{-1} - I$ can be used in stead of the eigenvalues of $\eta V_{cs}V_p^{-1} - I$ to obtain sufficient conditions for a pure panel or a pure cross section to be optimal. Because the minimal eigenvalue of the first matrix is not smaller than the minimal eigenvalue of the latter and analogously the maximal eigenvalue of the first matrix is bounded by the maximal eigenvalue of the latter, the sufficient conditions for optimality of pure panels or cross sections obtained above will still be sufficient in case of linear restrictions on the parameters.

In order to illustrate these results we consider the estimation of the marginal budget shares of the consumption categories food and clothing assuming that (2) is valid where y_{it} denotes the expenditures on one of the two consumption categories and x_{it} denotes total expenditures on non-durables. The model can be motivated by a two stage budgetting argument where the total expenditures on non-durables in every month are determined prior to the decision on how to split them over the various categories.

The maximum likelihood estimates of ρ in (2) for food and clothing are .74 and .16 with standard errors .005 and .001 respectively. The LM test statistics against first order autocorrelation in the ε_{it}'s introduced in section three equal .41 and 3.73 respectively. If (14) is imposed on the expenditures on non-durables the ML estimate of ρ_x is .41 with standard error .003.

These estimates of ρ and ρ_x suggest that a pure panel will be optimal for every linear combination of the marginal budget shares if $\eta > .47$ for food and $\eta > 1.23$ for clothing. Cross sections are optimal if $\eta < .26$ and $\eta < .84$ respectively. If the actual value of η is somewhere between 1.3 and 1.7 as suggested in the introduction these results of course imply that if the aim is to analyse marginal budget shares, unlike the results in section three on the estimation of period means, the optimal design does not depend on the linear combination of the parameters one is interested in.

The LM test statistic against first order autocorrelation in ε_{xit} in (14) takes the insignificant value of .10. The largest unrestricted ML estimate of Ω_{ts} ($t \neq s$) is however .93. If this value is used the conditions for optimality of a pure panel will change into $\eta > 2.86$ and $\eta > 2.41$ for food and clothing respectively. Note however that (16) yields only a rough bound of the largest eigenvalue of Ω. If this eigenvalue is estimated directly the bounds for optimality will reduce to .51 and 1.29 respectively, which again imply optimality of a panel design. Using the minimal eigenvalue to obtain upper bounds of η for a cross section to be optimal yields $\eta < .26$ and $\eta < .89$.

5. Summary

In this note we derived a number of simple conditions which can be used to asses whether a panel or a cross section or a combination of both will yield most efficient estimates of some linear combination of time dependent parameters in a linear model. These results can be generalized in a straightforward manner to other models.

In the empirical analysis it was shown that if one is estimating period means, it will often unfortunately strongly depend on the linear combination of the time means to be estimated which type of data will be preferable. If an exogenous variable with a relatively small individual effect, such as total expenditures on non-durables is included in the model the optimal design for the estimation of the regression coefficients will be somewhat simpler to obtain.

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Appendix

Derivation of the LM test statistic

The Lagrange Multiplier test against first order autocorrelation in the ε_{it} in (1) is the test against the alternative

$$y_{it} = \mu_t + \alpha_i + u_{it}$$
$$u_{it} = \gamma\, u_{i,t-1} + \varepsilon_{it}$$

where $V\{\alpha_i + u_i\} = \Omega = \sigma_\varepsilon^2 \begin{bmatrix} 1 & \gamma & \gamma^2 \ldots & \gamma^{T-1} \\ \gamma & 1 & \gamma \ldots & \vdots \\ & & & \vdots \\ \gamma^{T-1} & \ldots\ldots & \gamma & 1 \end{bmatrix} + \sigma_\alpha^2\, \iota_T \iota_T'$.

The null hypothesis is H_0: $\gamma = 0$ and the loglikelihood is given by

$$L = \sum_{i=1}^{N} L_i = \text{constant} - \frac{1}{2}\sum_{i=1}^{N} \log|\Omega| - \frac{1}{2}\sum_{i=1}^{N} (y_i-\mu)'\Omega^{-1}(y_i-\mu).$$

Let $\psi' = (\psi_1, \psi_2, \psi_3) = (\sigma_\varepsilon^2, \sigma_\alpha^2, \gamma)$; then

$$\frac{\partial L_i}{\partial \psi_k} = -\frac{1}{2}\sum_{t=1}^{T}\sum_{s=1}^{T} \frac{\partial \omega_{ts}}{\partial \psi_k}.\frac{\partial \log|\Omega|}{\partial \omega_{ts}} - \frac{1}{2}\sum_{t=1}^{T}\sum_{s=1}^{T} (y_i-\mu)'\frac{\partial \omega^{ts}}{\partial \psi_k}(y_i-\mu),$$

where ω_{ts} and ω^{ts} are the (t,s)-elements of Ω and Ω^{-1} respectively.
Using

$$\frac{\partial \log|\Omega|}{\partial \omega_{ts}} = \omega^{ts}$$

and

$$\frac{\partial \Omega^{-1}}{\partial \psi_k} = -\Omega^{-1}\frac{\partial \Omega}{\partial \psi_k}\Omega^{-1}$$

we can write

$$\frac{\partial L_i}{\partial \psi_k} = -\frac{1}{2}\,\text{trace}\left[\frac{\partial \Omega}{\partial \psi_k}\Omega^{-1}\right] + \frac{1}{2}(y_i-\mu)'\Omega^{-1}\frac{\partial \Omega}{\partial \psi_k}\Omega^{-1}(y_i-\mu),.$$

which is straigthforward to compute under the null.
Since the Fisher information matrix is block diagonal with respect to ψ and μ, the LM test statistic for $\gamma = 0$ can be written as (see e.g. Engle [1984])

$$\xi_{LM} = (\Sigma\frac{\partial L_i}{\partial \psi'})\;(\Sigma\frac{\partial L_i}{\partial \psi}\frac{\partial L_i}{\partial \psi'})^{-1}\;(\Sigma\frac{\partial L_i}{\partial \psi}),$$

to be evaluated under H_0. Consequently ξ_{LM} can be calculated as N times the non-centered R^2 of a regression of ι_N on $\frac{\partial L_i}{\partial \psi_k}$ (k = 1, 2, 3). As is well known, under the null hypothesis ξ_{LM} converges in distribution to a central χ^2 distribution with one degree of freedom. The test against autocorrelation in (2) can be derived along similar lines.

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